Estimation of Mid-Air Collision Risk in High-Density Airspace Using Hexagonal Spatial Indexing

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Abstract: Civil aviation safety remains a foundational element of the modern air transport system. Each of air transportation services must be delivered in compliance with the minimum required safety levels. Contemporary safety assessment frameworks typically integrate the cumulative impact of hazardous factors on the nominal functioning of the air transport system, with detailed analyses often employing tree-structured models of risk propagation. Among all operational hazards, the risk of mid-air collision is one of the most critical, particularly given the sustained global growth in air traffic demand. Increasing aircraft density within constrained airspace volumes requires new analytical methods capable of supporting both safety assurance and efficient airspace utilization. This paper presents a comparative study of two collision-risk models suitable for airspace safety analysis. The first model explicitly incorporates three-dimensional airspace volume, while the second aggregates risk across vertical and horizontal planes. To enhance computational scalability, a hierarchical hexagonal spatial indexing system is applied for the rapid identification of potentially conflicting aircraft pairs. The resulting hybrid framework provides high-speed and accurate detection of potential conflicts, making it a valuable instrument for the modernization of air traffic management, particularly in increasingly complex environments involving both manned and unmanned aircraft. The proposed methodology is validated using ADS-B observational data from German airspace.

Keywords: Air Navigation, Air Traffic Management, Collision Risk, Intelligent Transport Systems, Spatial Indexing.

1. INTRODUCTION

Mid-air collision is one of the most hazardous events in civil aviation. Its critical importance arises from the involvement of at least two aircraft and the potentially catastrophic consequences [1, 2]. Many safety manuals classify mid-air collisions as among the most severe aviation accidents. However, civil aviation regulations define a mid-air collision more broadly, as any violation of prescribed safety boundaries between aircraft, regardless of whether physical contact occurs [3]. These safety boundaries are established through minimum separation standards in both horizontal and vertical planes to ensure a safe distance between airspace users. Thus, any infringement of minimum separation can be considered a mid-air collision event, even in the absence of an actual impact.

To mitigate these risks, modern air transportation system relies on a range of onboard and ground-based technologies. Heavy aircraft are equipped with the Airborne Collision Avoidance System (ACAS), which monitors surrounding traffic, identifies situations where separation minima may be violated, and issues coordinated advisories to pilots for vertical maneuvers to avoid potential collisions [4]. In parallel, air traffic service providers operate surveillance data processing systems that contain conflict-detection algorithms to identify developing hazardous situations and alert air traffic controllers [5]. These onboard and ground systems operate independently yet complementarily to ensure collision-free traffic flows.

Global air traffic continues to grow annually. The advantages of air transport such as speed, convenience, and global connectivity drive a steady increase in the number of aircraft operating simultaneously within limited airspace. This growth places pressure on available airspace capacity, contributing to delays and potentially increasing the risk of deviations from nominal operations.

Moreover, during the last decade airspace users face significant challenges connected with limit of available airspace caused by multiple military conflicts around the globe. The war in Ukraine and conflicts in the Middle East have caused serious problems to safety of air transportation [6]. This also forces airspace users to adopt detour trajectories to avoid dangerous zones, which significantly increases the load on neighboring airspaces adjacent to the closed ones.

Another important development is the integration of unmanned aerial vehicles into controlled airspace. Concepts such as U-space and Urban Air Mobility illustrate the rapid expansion of unmanned aerial vehicles operations, which will substantially increase the number of airspace users and the overall complexity of traffic management [7-9].

Collectively, these factors of growth in conventional air traffic, reduction of available airspace, and integration of unmanned aerial vehicles operations can

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exacerbate the risk of mid-air collision, making renewed research into collision risk assessment more relevant than ever.

This article presents the results of a mid-air collision risk assessment conducted for a specific region of airspace. The proposed algorithm identifies the potential for mid-air collisions by evaluating risk values within a geographically limited area. The method is based on a hybrid collision-risk model adapted for free-route airspace and integrated into an automated air traffic management framework built on a global spatial indexing system. The airspace is represented as a set of hexagonal cells, enabling the estimation of collision risk for each elementary spatial segment. This structure is implemented using the Python H3 library [10, 11], which supports both the spatial representation and the visualization of collision-risk distribution within the study region.

2. AIRSPACE AND AIR TRAFFIC MANAGEMENT

Collision risk models in aviation are mathematical and statistical tools used to estimate the probability of collision between aircraft. Their main purpose is to ensure flight safety, optimize separation (vertical and horizontal separation of aircraft in airspace), and support decision-making by controllers and automated control systems [12].

Such models are used in civil and military aviation, as well as for unmanned aerial vehicles. They are used in route planning, developing safety standards, certifying new aviation systems, and for incident analysis and accident investigation.

There are several types of collision risk models. The most common are analytical models based on mathematical formulas, and simulation models that simulate real flights in a virtual environment. Without such models, contemporary approaches—such as machine learning, heuristic algorithms, and artificial intelligence systems—are being used more frequently. They can analyze large amounts of data, predict dangerous situations, and autonomously make decisions about changes.

Based on these models, the aviation industry can effectively manage air traffic, reduce risks, and increase the overall level of flight safety.

There are various approaches to representing an aircraft in collision-risk modeling. Due to the complex geometry and proportions of real aircraft, it is impractical to construct an exact geometric mask suitable for analytical calculations. For this reason, simplified geometric shapes, such as cylinders or rectangular boxes are commonly used. Their selection is not arbitrary, ICAO guidance indicates that box-shaped representations are more appropriate for modeling parallel routes, while cylindrical representations are better suited for intersecting routes. These simplified forms significantly reduce computational complexity while preserving the essential characteristics relevant to collision risk.

In the standard collision model, the probability of collision is computed by evaluating the likelihood that the protected volumes of different aircraft intersect or overlap. Each potential aircraft pair is analyzed individually to determine whether their simplified geometric representations result in a conflict.

The dimensions of these same sectors can be considered either as double the dimensions of the aircraft itself (for example, its length) or as double the unit of measurement of the minimum separation (time or distance). This is primarily due to the accuracy of the navigation aids available in the sector. One controller, as the main person responsible for airspace safety and directing air traffic in such a way as to avoid collisions, operates on the basis of data obtained from radars, which, like any technical device, have the value of accuracy, which is inherent in the minimum dimensions in the process of the latest development of technology observed in aviation, the essence has evolved from visual control to the use of primary (PSR) and secondary (SSR) surveillance radars, as well as automatic dependent surveillance with broadcast (ADS-B). This system is the technological foundation of modern navigation concepts, in particular Area Navigation (RNAV) and Performance Based Navigation (PBN). The PBN framework requires the maintenance of strict accuracy standards, such as keeping on track with accuracy of at least 95% of the time, and its Requirements for Navigation Performance (RNP) component adds mandatory performance monitoring and notification of deviations. The maintenance of agreed principles of airspace organization by ICAO member states provides a single global air traffic management (ATM) system to unify rules, optimize resources and reduce the risk of conflicts.

However, the implementation of the concept of Free Route Airspace (FRA), which gives airlines greater flexibility in route selection, complicates traffic forecasting and increases the workload on controllers.

In these conditions, traditional methods of collision risk assessment become insufficient [13]. Effective risk management in the context of a changing navigation reality requires the adaptation of: accurate spatial separation, flexible assessment models and automated visualization tools to support relevant decisions in real time.

3. METHODOLOGY

Several methodological approaches exist for calculating mid-air collision risk, each based on different theoretical principles. The most common and widely used, particularly for transatlantic operations and parallel-route structures, is the Reich model, which also forms the basis of ICAO's standardized collision-risk calculations [3]. In this model, each aircraft is surrounded by a rectangular safety zone, represented as a parallelepiped whose dimensions correspond to the horizontal and vertical separation minima applicable to the airspace. A violation occurs when two such safety volumes overlap, indicating a loss of prescribed separation and the potential onset of a hazardous situation.

Beyond the Reich model, several other models have been developed to address more complex or less structured traffic geometries [14, 15]. The Anderson-Hsu model combines the strengths of the earlier Hsu and Anderson formulations. Unlike the Reich model, best suited for parallel operations is Anderson-Hsu incorporates detailed calculations for intersecting routes, explicitly accounting for navigation and positioning errors. The original Hsu model was designed to characterize risk in crossing-route scenarios, while the Anderson model focused more broadly on total system-wide collision risk; their integration enables a more robust assessment across diverse traffic patterns.

A further modification of this approach is the Aldis model, which also targets intersecting-route scenarios but extends the analysis by incorporating the statistical distributions of aircraft speeds and inter-arrival times. This enhancement makes it possible to estimate more accurately the probability that two aircraft will reach a conflict point simultaneously and with insufficient separation yielding a more realistic assessment of collision likelihood in dynamic environments.

In contrast to these route-based models, the Gas model adopts a fundamentally different conceptual basis. It draws an analogy from the free movement of gas molecules and calculates collision risk not from geometric overlaps of safety zones but from the fraction of a defined airspace volume occupied by moving aircraft. This approach is particularly useful for evaluating risk in unstructured or highly dense airspace, where traditional route concepts are weak or absent, such as operations involving large numbers of unmanned aircraft systems or within FRA. Although FRA retains vertical flight levels, its horizontal structure is far less constrained, making gas-based modeling a valuable tool for assessing overall system throughput and safety.

The proposed methodology is based on a hybrid collision-risk assessment model that incorporates both the geometric characteristics of conflict pairs (trajectory intersection angle, convergence rate, vertical separation) and the statistical properties of traffic within the considered FIR. The model combines classical approaches used for parallel and crossing traffic with elements of Markov processes to capture the dynamics of aircraft transitions between sectors, as well as gas-model principles to estimate local traffic density.

A hierarchical hexagonal spatial indexing system is implemented using the H3 is employed to represent airspace. This structure partitions the region into equal-area hexagonal cells, ensuring uniform data aggregation and reducing local estimation errors. Each cell stores information on the number of aircraft passing through it, their trajectories, speeds, altitudes, and the type of conflict interaction observed.

The collision-risk assessment is performed in several stages:

1. Spatial aggregation of ADS-B data. For each airspace cell, the system identifies the number of unique airplanes, mean convergence speed, vertical separation, and the frequency of conflict situations.

2. Classification of conflict pairs. Clustering algorithms (such as DBSCAN or HDBSCAN) are used to identify groups of aircraft with potentially hazardous trajectories and similar dynamical behavior.

3. Probability estimation of mid-air collision. A combined risk model is applied, integrating geometric parameters with local traffic density metrics derived from the hexagonal segmentation.

4. Risk visualization. The results are rendered in the form of a risk map, where each H3 cell is assigned a numerical risk score. This format enables rapid identification of high-risk airspace segments and facilitates tactical decision-making.

Special emphasis is placed on the influence of onboard navigation system accuracy on collision risk. The model incorporates positioning errors, coordinate-update delays, and GNSS signal quality or factors that significantly affect the reliability of conflict detection and prediction.

To automate the assessment process, a Python-based software module has been developed, integrated with air traffic management systems. The module supports periodic, near-real-time recalculation of risk values, enabling continuous adaptation to evolving traffic conditions and operational constraints. This approach not only identifies high-risk regions but also provides actionable recommendations for route optimization and control prioritization.

The gas collision model is a statistical approach in air traffic control. It models aircraft as gas particles moving along random trajectories in 3D space. This method allows predicting the probability of conflicts or emergencies among aircraft, which is critically important in conditions of high traffic density and in the development of automatic control systems [16, 17]:

$$\mathrm{C} = \frac{\mathrm{N}^2}{2\mathrm{B}}\left(\pi \mathrm{g}^2 \mathrm{E}(|\mathrm{V}_{\mathrm{rv}}|) + 5\mathrm{ghE}(|\mathrm{V}_{\mathrm{rh}}|)\right), \quad (1)$$

where B is airspace volume; g is horizontal dimensions of the aircraft; h is vertical dimensions of aircraft; $E(|V_{rv}|)$ is expected vertical relative velocity; $E(|V_{rh}|)$ is expected horizontal relative velocity.

According to this model, an aircraft is represented as a geometric body (cylinder) moving at a specific speed, direction, and altitude. A collision is recorded as a geometric coincidence or superposition of these model objects.

Expected horizontal relative velocity is calculated based on horizontal velocities of airplanes involved in conflict:

$$E(|V_{rh}|) = \frac{1}{2\pi}\int_0^{2\pi} \sqrt{(V_1{}^2 + V_2{}^2 - 2V_1V_2\cos\beta)}\, d\beta \quad (2)$$

where V_1 and V_2 are the horizontal velocities of airplanes; β is angle between the flight directions of airplanes.

The proposed methodology goes beyond assessing the probability of actual mid-air collisions and is successfully adapted to identify hazardous proximities at an early stage. This preventive analysis significantly enhances safety, as it enables the detection of dangerous approaches before they escalate into critical situations. A key advantage of the method is its computational simplicity: evaluating only the vertical speed components of aircraft is sufficient to determine whether they are converging or diverging, allowing rapid screening of potentially unsafe encounters.

The study relies on real-world data obtained from open ADS-B broadcasts, ensuring high fidelity of analysis. The positional accuracy of these signals is comparable to that provided by modern onboard navigation systems. Access to a large volume of high-quality surveillance data is achieved through the widely recognized OpenSky Network platform [18].

To automate the assessment process, a flexible Python-based analytical framework has been developed. This system is capable of storing, processing, and continuously analyzing air traffic data. The most innovative element of the approach is the application of hexagonal spatial indexing (H3). Instead of working directly with raw geographic coordinates, airspace is discretized into uniform hexagonal cells. This spatial representation dramatically reduces computational complexity and enables the system to adapt to rapidly changing traffic patterns in near real time.

Overall, the methodology provides scalable, high-resolution risk models that can be efficiently integrated into next-generation automated air traffic management systems. By enabling early identification of hazardous encounters and providing a robust computational framework, the proposed approach contributes to improving the safety, resilience, and efficiency of modern airspace operations.

4. NUMERICAL VALIDATION

To illustrate the results of the study, simulated and accumulated air traffic data from ADS-B were used to assess mid-air collision risk within the German airspace. The dataset covers all recorded traffic between 7 January 2025 and 18 January 2025, comprising over 15 million positional updates. Data collection and processing were carried out using a Python-based software framework that included custom algorithms for risk modeling.

Initially, aircraft positions were selected within a rectangular bounding box using the *get_states()* function. This initial selection served as a pre-filter to optimize processing time. A geometric mask was then applied to filter trajectories precisely within the FIR (Flight Information Region) boundaries, ensuring analysis was strictly focused on defined German airspace. Full aircraft trajectories were reconstructed using unique ICAO identifiers with the *get_track_by_aircraft()* function. Subsequently, a probabilistic conflict detection model was applied to these reconstructed tracks to quantify collision risk.

The resulting dataset includes 397 unique aircraft representing 38 different aircraft types. The most common types were the Airbus A320 (38 aircraft), A319 (23 aircraft), and A20N (18 aircraft), while the remaining types accounted for fewer than 15 aircraft each. Aircraft-specific and geometric parameters, such as fuselage length and wingspan, were obtained from the Base of Aircraft Data model developed by EUROCONTROL. These parameters were essential for calculating the kinematic properties of aircraft within the risk assessment model. An example of the input dataset and the derived values is presented in Table **1**.

The data given in Table **1** indicate the calculated values from formula (1) where πg2 is the area of the horizontal image of the aircraft, which according to the formula will be a circle with a diameter of 2g, and 5gh is the area of the vertical image of the aircraft in the form of a rectangle increased by a factor of 5. These values are needed so that after multiplying by the corresponding speeds we get the sum of the volume of air that the aircraft occupies per unit of time.

A hierarchical hexagonal indexing system is used to intelligently divide the airspace into a grid of regular cells. Aircraft coordinates obtained from the Global Navigation Satellite System (GNSS) are used to establish the nearest hexagonal cell. The 6th accuracy level H3 is used to account for the level of GNSS position measurement error and the minimum permissible horizontal separation. A hexagonal cell of the 6th accuracy level has an average edge length of approximately 3.7 km. This value is comparable to the minimum established horizontal separation between flight levels (5.6 km). The edge length is equivalent to the radius of the circle describing the cell, resulting in a cell diameter of approximately 7.4 km. Which is more than the minimum horizontal separation in the airspace specified above, due to this, in the event of a conflict situation within the hexagon, we will understand that there is most likely no minimum separation between the aircraft. Compared to other types of indexing, hexagons have a number of significant advantages:

- firstly, they do not leave empty cells, which minimizes the chance of losing the aircraft's position and reduces the chance of an error in determining the aircraft's position.
- secondly, the distance to all neighboring cells from the center of the main cell is the same,

Table 1: Planes and Quantities for Formula (1)

Unique airplane code used in ADS-B	Horizontal area of an aircraft mask (πg^2),m^2	Vertical area of the aircraft mask (5gh),m^2
4d2014	4417.86	2193.75
44ce62	3589.08	6337.5
407b54	6221.14	2603.25
4b15ed	4417.86	8343.75
3c48f1	3589.08	1977.3
3c6590	3589.08	5712.2
3c6583	3589.08	1977.3
440cab	4417.86	6337.5
3c56ef	4417.86	2193.75
3c6750	4417.86	7031.25
3c65cb	4417.86	2193.75
3c65c8	4417.86	7031.25
4a08ec	4901.67	2488.5
3c5b31	12707.62	12561.0

which greatly simplifies mathematical calculations.

- thirdly, hexagonal cells, which have approximately the same area, allow for an accurate and unambiguous assessment of the local density of traffic, population, or other parameters.

Knowing the location of all aircraft in cells allows us to quickly sort them, finding aircraft that are simultaneously within the same hexagon.

To visualize the results, the folium library in Python was used to overlay the processed data on OpenStreetMap. The spatial picture of air traffic intensity in the study region is shown in Figure **1**.

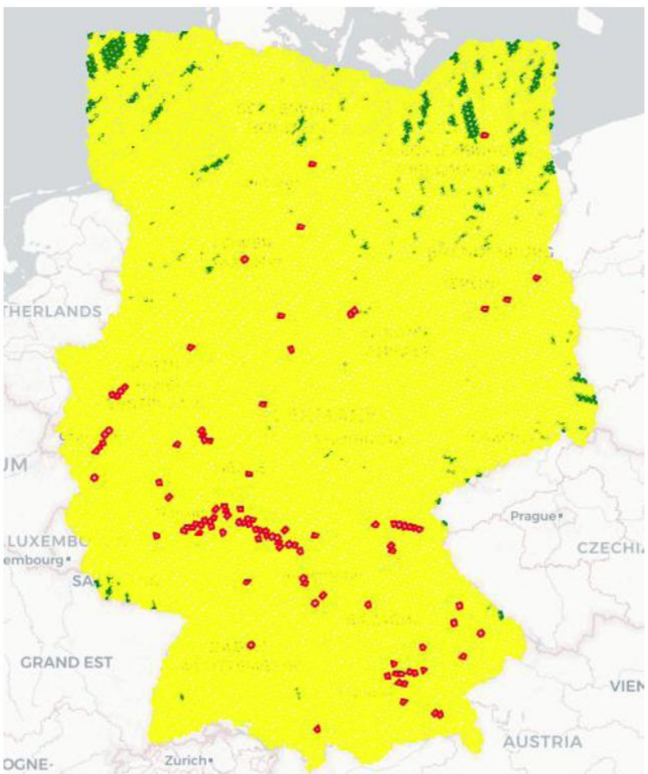

Figure 1: Amount of airspace users within unique cell, over the entire study period.

Regions where no movement was observed during the analysis period are marked in Figure **1** in green, while hexagonal cells demonstrating the simultaneous presence of two or more aircraft are highlighted in red. Areas with the activity of only one aircraft without temporal overlap are marked in yellow. This visualization method allows for a clear assessment of air traffic density and helps identify areas of high intensity that may correspond to an increased risk of collision. The implementation of this H3 system significantly increases the efficiency and accuracy of airspace monitoring. Due to its high computational efficiency, it is a powerful tool for automating air traffic control processes and minimizing potential threats to flight safety in real time.

The identified pairs of aircraft, located in unique hexagonal cells, are used in the risk analysis based on model (1) to estimate the probability of mid-air collision. Risks are calculated pairwise according to formula (1), we have speed, angle, we have the aircraft parameters from Table **1**. The airspace volume is calculated as the product of the maximum area of a hierarchical hexagon of dimension 6, which is 43.59 km^2, and the altitude of echelon 660 or 20.1 km. The results of the risk assessment are given in Table **2** for the eight identified pairs of airspace users based on the highest risk value. It also shows between which aircraft and during which flights, when and in which hexagon the risk assessment was carried out.

Results of air traffic study show that the highest value of mid-air collision risk (2.667e-06) was identified in pair of airplanes 44ce62 and 4d2014 within cell of hexagon 861fa535fffffff on 2025-01-11 14:26:05 UTC .The lowest risk of mid-air collision for pair within hexagonal cell 861f8d0efffffff was 3.232e-08 for 3c65c8 and 3c65cb has identified on 2025-01-09 22:06:00 UTC. All the hexagons given in Table **2** are highlighted in Figure **2**. The saturation of the red color corresponds to the magnitude of the risk.

To understand how large or small our results are, we will use the Target Safety Level (TLS). It is used in civil aviation to determine the minimum level of risk required to ensure flight safety. TLS = 5×10^{-9} is used as the maximum acceptable risk in civil aviation. Accordingly, all results above this are a sign of danger, and all results below mean that the situation is safe.

As can be seen from Table **2**, most of the risk values are in the range $(1\text{-}3)\times10^{-6}$. In fact, all data exceed the specified TLS level. This is directly related to the fact that model (1) has a fairly simple formula for calculation. It contains a direct ratio of the volume of airspace that the aircraft overcomes per second to the total volume of airspace that we have, respectively, since the numerator has 2 dynamic indicators that change over time, namely the relative speeds vertically and horizontally, it can be assumed that achieving lower risks is possible by reducing them. However, additional calculations showed that achieving optimal indicators is possible at a relative speed of less than 50

Table 2: Planes and Quantities for Formula (1)

Unique index of hexagonal cell	User A identification code	User B identification code	Time, UTC	Risk in Gas model
861fa535fffffff	44ce62	4d2014	2025-01-11 14:26:05	2.667e-06
861fa929fffffff	4b15ed	407b54	2025-01-09 22:15:04	2.436e-06
861f8d0f7ffffff	3c6590	3c48f1	2025-01-14 21:05:24	2.381e-06
861faea87ffffff	440cab	3c6583	2025-01-08 22:30:40	1.786e-06
861f8d0f7ffffff	3c6750	3c56ef	2025-01-10 22:21:31	1.122e-06
861f8d0efffffff	3c65c8	3c65cb	2025-01-09 22:05:59	9.62e-07
861f8daafffffff	3c5b31	4a08ec	2025-01-14 21:10:52	2.06e-07
861fa8717ffffff	44ce62	400a90	2025-01-08 17:39:38	1.89e-07

m/s, which was not observed during the study where the main indicators ranged from 300 to 500 m/s. It is possible to reduce the risks shown in Table **2** by increasing the selected volume of the studied space, which is denoted in the denominator of model (1) as B. As noted earlier, we use a hexagonal hierarchical system to determine risks, and accordingly, the size of the studied airspace used in model (1) will correspond to the size of the volume of the hexagon we have chosen, the horizontal parameters of which we know(43.59 km^2), and the vertical ones will be 20116 m, i.e. 660 FL in aviation (the most extreme at which general aviation aircraft are allowed to fly), the total volume is 0.86 × $10^{13}m^3$, i.e. if we choose a larger hexagon (larger horizontal area), we will get a larger volume in the denominator, which will lead to a decrease in the risk value.

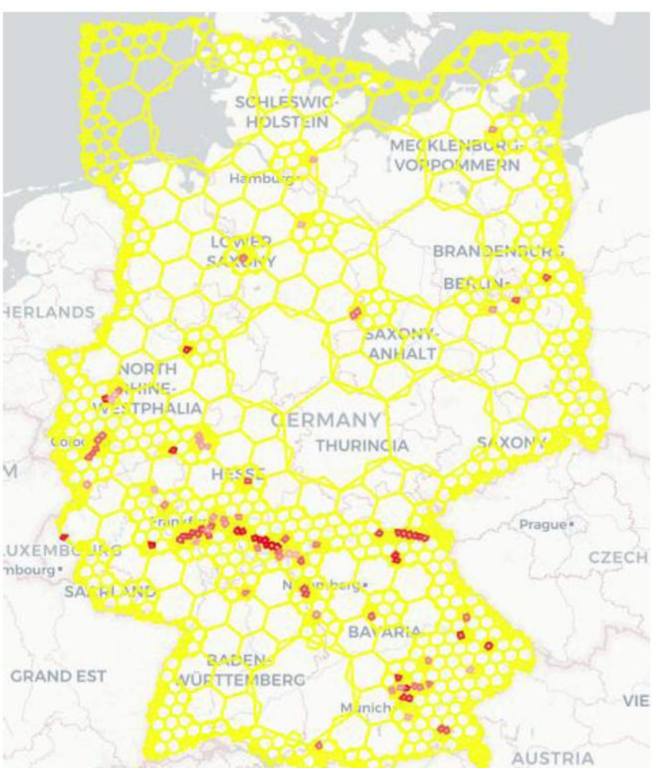

Figure 2: Distribution of risk intensity in the studied airspace.

5. CONCLUSIONS

Ensuring flight safety and avoiding mid-air collisions remains a key task of modern aviation, which is ensured by the constant increase in air traffic intensity and the integration of unmanned systems into controlled space. In this work, a new approach was proposed in using the collision risk assessment model (Gas model) supplemented by a hierarchical system of the hexagonal H3 index. This combination allowed achieving high data processing speed, reducing local estimation errors, and ensuring the scalability of models for complex air traffic scenarios.

The main results of the study confirmed that:

- the use of H3-indexing provides uniform data aggregation and effective detection of conflicting pairs of aircraft in real time;
- the model allows estimating traffic density in a selected location at a selected time, which increases the accuracy of risk assessment;
- the use of ADS-B data from open sources (OpenSky Network) provides high reliability of the analysis and practical applicability of the methodology for integration into modern air traffic control systems.

The new contribution of the work is created in the created complex tool, which consists of traditional risk models with modern spatial indexing methods, which allows not only to assess the probability of actual collisions, but also to detect dangerous approaches at early stages. This is a high level of preventive safety and opens up opportunities for automation of monitoring processes.

Next steps for future research

- expansion of the model for multi-level analysis, taking into account different types of air users (commercial aviation, drones, military aircraft);
- integration of machine learning algorithms for risk prediction based on historical data and detection of hidden patterns;
- adaptation of the methodology to Free Route Airspace conditions and urban environments with a high density of unmanned aircraft;
- development of interfaces for operational use of the results in air traffic control systems and support for controller decisions.

Thus, the proposed methodology demonstrates significant potential for modernizing the flight safety system, and its further development can become the basis for creating new risk assessment standards in global aviation practice.

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